Spiral plat avec courbe terminale concentrique

Déformée élastique en position horizontale

Cas d'une montre bracelet

Caractéristiques du spiral

Référence :E:\Résonateur (TA)\Data\Bal_spiral plat (ex num).mcd(R)

Référence :E:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $ép = 0.03\,mm$ $ha = 0.15\,mm$ $S = 4.5\times10^{-3}\,mm^2$ $TOL := 10^{-9}$

$d2_{sp} = 4.52\,mm$ $d1_{sp} = 1.1\,mm$ $d_{piton} = 5.1\,mm$ $p_{sp} = 0.135\,mm$ $n_{sp} = 12.667$

$L := L_{sp}$ $L = 11.182\,cm$ $\psi_0 := 2\cdot\pi\cdot n_{sp}$ $\psi_0 = 4.56\times10^3\,deg$

Position des goupilles de raquettes $r_{GR} := 0.5\cdot d_{piton}$ $\alpha_{GR} := 0$ $x_{GR} := r_{GR}\cdot cos(\alpha_{GR})$ $y_{GR} := r_{GR}\cdot sin(\alpha_{GR})$

$x_{GR} = 2.55\,mm$ $y_{GR} = 0\,mm$ $z_{GR} := x_{GR} + i\cdot y_{GR}$

Position du point de raccordement sur le spiral $\alpha_A := 30\cdot deg$ $r_A := 0.5\cdot d2_{sp}$ $z_A := r_A\cdot e^{i\cdot\alpha_A}$

Courbe terminale $r_t := r_{GR}$ $x_{0t}(\alpha_t) := r_t\cdot cos(\alpha_t)$ $y_{0t}(\alpha_t) := r_t\cdot sin(\alpha_t)$ $z_{0t}(\alpha_t) := r_t\cdot e^{i\cdot\alpha_t}$

$s_t(\alpha_t) := r_t\cdot\alpha_t$ $l_t := s_t(\alpha_A)$ $l_t = 1.335\,mm$

Position du point d'attache à la virole $r_V := 0.5\cdot d1_{sp}$ $\alpha_V(\theta) := \psi_0 + \alpha_A + \theta$ $x_V(\theta) := r_V\cdot cos(\alpha_V(\theta))$ $y_V(\theta) := r_V\cdot sin(\alpha_V(\theta))$

Forme initiale du spiral

$a := \dfrac{p_{sp}}{2\cdot\pi}$ $r_s(\alpha) := r_A - a\cdot(\alpha-\alpha_A)$ $x_{0s}(\alpha) := r_s(\alpha)\cdot cos(\alpha)$ $y_{0s}(\alpha) := r_s(\alpha)\cdot sin(\alpha)$

$z_{0s}(\alpha) := r_s(\alpha)\cdot exp(i\cdot\alpha)$

$s(\alpha) := \dfrac{1}{2\cdot a}\cdot\left(r_A^2 - r_s(\alpha)^2\right)$ $s(\alpha) := r_A\cdot(\alpha-\alpha_A) - \dfrac{a}{2}\cdot(\alpha-\alpha_A)^2$ $L_t := s(\psi_0+\alpha_A) + l_t$

$L_t = 11.315\,cm$

Amplitude stationnaire du balancier $\theta_0 = 270\,deg$

Contrainte maximum

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$I_{33} := I_{f\_rect}(ép, ha)$ $W_{f3} := W_{f\_rect}(ép, ha)$ $\sigma_{max} := \dfrac{E\cdot I_{33}}{L\cdot W_{f3}}\cdot\theta_0$ $\sigma_{max} = 132.293\,\dfrac{N}{mm^2}$

Déformée du spiral avec la virole non liée à l'axe de balancier

Courbe terminale externe

$\varphi_{0t1}(\alpha_t) := \alpha_t + \dfrac{\pi}{2}$ $z_{1t}(\theta,\alpha_t) := z_{GR} + r_t\cdot\displaystyle\int_{\alpha_{GR}}^{\alpha_t} i\cdot e^{i\cdot\alpha'_t}\cdot exp\left(i\cdot\frac{\theta}{L_t}\cdot r_t\cdot\alpha'_t\right)d\alpha'_t$

$z_{1t}(\theta,\alpha_t) := z_{GR} + \dfrac{L_t\cdot r_t}{L_t+\theta\cdot r_t}\cdot\left(exp\left(i\cdot\alpha_t\cdot\dfrac{L_t+\theta\cdot r_t}{L_t}\right)-1\right)$ $\Delta z_{1A}(\theta) := z_{1t}(\theta,\alpha_A) - z_{0t}(\alpha_A)$

Partie spiralée

$s'(\alpha) := r_A - a\cdot(\alpha - \alpha_A)$ $\qquad z'_0(\alpha) := \left[-a + i\cdot\left[r_A - a\cdot(\alpha - \alpha_A)\right]\right]\cdot exp(i\cdot\alpha)$ $\qquad \Delta\varphi_{1A}(\theta) := \theta\cdot\frac{l_t}{L_t}$

$\Delta z_{1s}(\theta,\alpha) := z_A + \int_{\alpha_A}^{\alpha} z'_0(\alpha')\cdot e^{i\cdot\Delta\varphi_{1A}(\theta)}\cdot exp\left(i\cdot\theta\cdot\frac{s(\alpha')}{L_t}\right)d\alpha'$ $\qquad z_{1s}(\theta,\alpha) := \Delta z_{1A}(\theta) + \Delta z_{1s}(\theta,\alpha)$

Graphe de la déformation

Forme naturelle

$n_t := 201$ $\quad j := 0..n_t - 1$ $\quad \Delta\alpha_t := \frac{\alpha_A - \alpha_{GR}}{n_t - 1}$ $\quad \alpha_{t_j} := \alpha_{GR} + j\cdot\Delta\alpha_t$ $\quad x_{t_j} := x_{0t}(\alpha_{t_j})$ $\quad y_{t_j} := y_{0t}(\alpha_{t_j})$

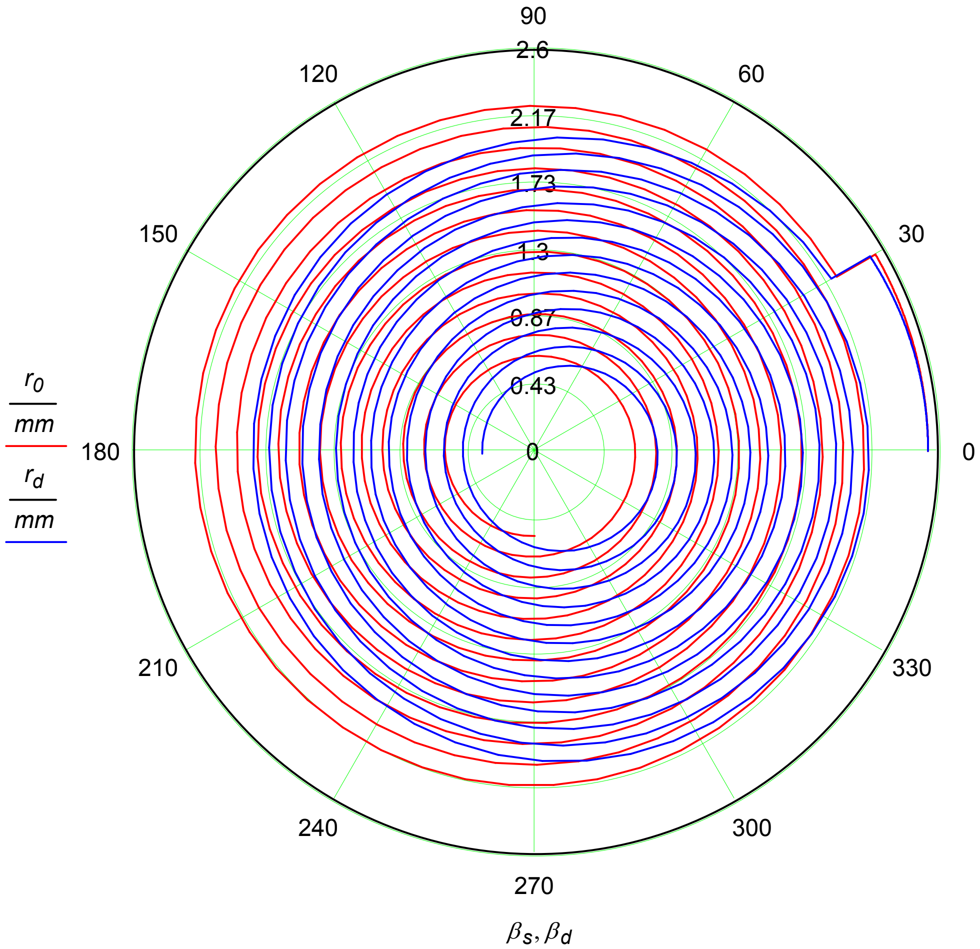
$n := 50\cdot partentière(n_{sp}) + 1$ $\quad i := 0..n - 1$ $\quad \Delta\alpha := \frac{\psi_0}{n-1}$ $\quad \alpha_i := i\cdot\Delta\alpha + \alpha_A$

$x_{s_i} := x_{0s}(\alpha_i)$ $\quad y_{s_i} := y_{0s}(\alpha_i)$ $\quad x_0 := pile(x_t, x_s)$ $\quad y_0 := pile(y_t, y_s)$ $\quad r_0 := \overrightarrow{\sqrt{x_0^{\,2} + y_0^{\,2}}}$ $\quad \beta_s := \overrightarrow{Atan(x_0, y_0)}$

Déformée $\quad z_{td_j} := z_{1t}(\theta_0, \alpha_{t_j})$ $\quad z_{sd_i} := z_{1s}(\theta_0, \alpha_i)$ $\quad z_d := pile(z_{td}, z_{sd})$

$n_{pt} := dernier(z_d)$ $\quad x_d := Re(z_d)$ $\quad y_d := Im(z_d)$ $\quad r_d := \overrightarrow{|z_d|}$ $\quad r_{d_{npt}} = 0.342\,mm$

$\beta_d := \overrightarrow{Atan(x_d, y_d)}$ $\quad \beta_{d_0} = 0\,deg$ $\quad \beta_{d_{npt}} = 182.931\,deg$ $\quad mod(\alpha_V(\theta_0), 2\cdot\pi) = 180\,deg$



$x_V(\theta_0) = -0.55\,mm$ $\quad x_{d_{npt}} - x_V(\theta_0) = 0.208\,mm$ $\quad y_V(\theta_0) = 0\,mm$ $\quad y_{d_{npt}} - y_V(\theta_0) = -0.017\,mm$

Déplacement de la virole libre

Contribution du spiral sans ses courbes terminales

$$s_s(\alpha) := s(\alpha) + l_t \qquad f(\theta,\alpha) := i\cdot\theta\cdot exp\left(i\cdot\theta\cdot\frac{s_s(\alpha)}{L_t}\right)$$

$$\mathbf{\Delta_s}(\theta) := \frac{1}{L_t}\cdot\int_{\alpha_A}^{\alpha_A+\psi_0} z_{0s}(\alpha)\cdot f(\theta,\alpha)\cdot s'(\alpha)\,d\alpha \qquad \mathbf{\Delta_s}(\theta_0) = -0.177 - 0.103i\,mm$$

Contribution de la courbe terminale externe

$$\mathbf{\Delta_t}(\theta) := \frac{i\cdot\theta}{L_t}\cdot r_t\cdot\int_0^{\alpha_A} z_{0t}(\alpha_t)\cdot exp\left(i\cdot\frac{\theta}{L_t}\cdot r_t\cdot\alpha_t\right)d\alpha_t \qquad \mathbf{\Delta_t}(\theta) := \frac{-\theta\cdot r_t^{\,2}}{L_t+\theta\cdot r_t}\cdot\left(1 - exp\left(i\cdot\alpha_A\cdot\frac{L_t+\theta\cdot r_t}{L_t}\right)\right)$$

$$\mathbf{\Delta_t}(\theta_0) = -0.04 + 0.134i\,mm$$

Contribution du spiral entier

$$\mathbf{\Delta_1}(\theta) := \mathbf{\Delta_t}(\theta) + \mathbf{\Delta_s}(\theta) \qquad \mathbf{\Delta_1}(\theta_0) = -0.217 + 0.031i\,mm$$

$$u_1(\theta) := Re(\mathbf{\Delta_1}(\theta)) \qquad v_1(\theta) := Im(\mathbf{\Delta_1}(\theta)) \qquad u_1(\theta_0) = -0.217\,mm \qquad v_1(\theta_0) = 0.031\,mm$$

Calcul des réactions

$$\xi_{0s} := \frac{1}{L_t}\cdot\left(\int_{\alpha_A}^{\alpha_A+\psi_0} x_{0s}(\alpha)\cdot r_s(\alpha)\,d\alpha + \int_0^{\alpha_A} x_{0t}(\alpha_t)\cdot r_t\,d\alpha_t\right) \qquad \xi_{0s} = 4.246\times10^{-3}\,mm$$

$$\eta_{0s} := \frac{1}{L_t}\cdot\left(\int_{\alpha_A}^{\alpha_A+\psi_0} y_{0s}(\alpha)\cdot r_s(\alpha)\,d\alpha + \int_0^{\alpha_A} y_{0t}(\alpha_t)\cdot r_t\,d\alpha_t\right) \qquad \eta_{0s} = 0.047\,mm$$

$$p2_{0s} := \frac{1}{L_t}\cdot\left(\int_{\alpha_A}^{\alpha_A+\psi_0} x_{0s}(\alpha)^2\cdot r_s(\alpha)\,d\alpha + \int_0^{\alpha_A} x_{0t}(\alpha_t)^2\cdot r_t\,d\alpha_t\right) \qquad p2_{0s} = 1.385\,mm^2$$

$$q2_{0s} := \frac{1}{L_t}\cdot\left(\int_{\alpha_A}^{\alpha_A+\psi_0} y_{0s}(\alpha)^2\cdot r_s(\alpha)\,d\alpha + \int_0^{\alpha_A} y_{0t}(\alpha_t)^2\cdot r_t\,d\alpha_t\right) \qquad q2_{0s} = 1.365\,mm^2$$

$$k_{0s} := \frac{1}{L_t}\cdot\left(\int_{\alpha_A}^{\alpha_A+\psi_0} x_{0s}(\alpha)\cdot y_{0s}(\alpha)\cdot r_s(\alpha)\,d\alpha + \int_0^{\alpha_A} x_{0t}(\alpha_t)\cdot y_{0t}(\alpha_t)\cdot r_t\,d\alpha_t\right) \qquad k_{0s} = 0.032\,mm^2$$

$$\mathbf{S_0} := \frac{L}{E\cdot I_{33}}\cdot\begin{pmatrix} q2_{0s} - \eta_{0s}^{\,2} & \eta_{0s}\cdot\xi_{0s} - k_{0s} \\ \eta_{0s}\cdot\xi_{0s} - k_{0s} & p2_{0s} - \xi_{0s}^{\,2} \end{pmatrix} \qquad \mathbf{R'}(\theta) := \mathbf{S_0}^{-1}\cdot\begin{pmatrix} u_1(\theta) \\ v_1(\theta) \end{pmatrix} \qquad \mathbf{R'}(\theta_0) = \begin{pmatrix} -1.001\times10^{-4} \\ 1.196\times10^{-5} \end{pmatrix}N$$

$$\left|\mathbf{R'}(\theta_0)\right| = 1.009\times10^{-4}\,N$$

Deuxième approximation de la déformée du spiral

$R'_x(\theta) := \mathbf{R}'(\theta)_0 \qquad R'_y(\theta) := \mathbf{R}'(\theta)_1$

$x_{1t}(\theta,\alpha) := Re(z_{1t}(\theta,\alpha)) \qquad y_{1t}(\theta,\alpha) := Im(z_{1t}(\theta,\alpha))$

$x_{1s}(\theta,\alpha) := Re(z_{1s}(\theta,\alpha)) \qquad y_{1s}(\theta,\alpha) := Im(z_{1s}(\theta,\alpha))$

$$s\xi_{1t}(\theta,\alpha_t) := \int_0^{\alpha_t} x_{1t}(\theta,\alpha'_t)\cdot r_t\, d\alpha'_t \qquad s\xi_{1s}(\theta,\alpha) := \int_{\alpha_A}^{\alpha} x_{1s}(\theta,\alpha')\cdot r_s(\alpha')\, d\alpha'$$

$$s\eta_{1t}(\theta,\alpha_t) := \int_0^{\alpha_t} y_{1t}(\theta,\alpha'_t)\cdot r_t\, d\alpha'_t \qquad s\eta_{1s}(\theta,\alpha) := \int_{\alpha_A}^{\alpha} y_{1s}(\theta,\alpha')\cdot r_s(\alpha')\, d\alpha'$$

$$\xi_1(\theta) := \frac{1}{L_t}\cdot\left(s\xi_{1t}(\theta,\alpha_A) + s\xi_{1s}(\theta,\psi_0+\alpha_A)\right) \qquad \eta_1(\theta) := \frac{1}{L_t}\cdot\left(s\eta_{1t}(\theta,\alpha_A) + s\eta_{1s}(\theta,\psi_0+\alpha_A)\right)$$

$$sp2_{1t}(\theta,\alpha_t) := \int_0^{\alpha_t} x_{1t}(\theta,\alpha'_t)^2\cdot r_t\, d\alpha'_t \qquad sp2_{1s}(\theta,\alpha) := \int_{\alpha_A}^{\alpha} x_{1s}(\theta,\alpha')^2\cdot r_s(\alpha')\, d\alpha'$$

$$sq2_{1t}(\theta,\alpha_t) := \int_0^{\alpha_t} y_{1t}(\theta,\alpha'_t)^2\cdot r_t\, d\alpha'_t \qquad sq2_{1s}(\theta,\alpha) := \int_{\alpha_A}^{\alpha} y_{1s}(\theta,\alpha')^2\cdot r_s(\alpha')\, d\alpha'$$

$$sk_{1t}(\theta,\alpha_t) := \int_0^{\alpha_t} x_{1t}(\theta,\alpha'_t)\cdot y_{1t}(\theta,\alpha'_t)\cdot r_t\, d\alpha'_t \qquad sk_{1s}(\theta,\alpha) := \int_{\alpha_A}^{\alpha} x_{1s}(\theta,\alpha')\cdot y_{1s}(\theta,\alpha')\cdot r_s(\alpha')\, d\alpha'$$

$$\mathbf{S_t}(\theta,\alpha_t) := \frac{1}{E\cdot I_{33}}\cdot\begin{bmatrix} -y_{1t}(\theta,\alpha_t)\cdot s\eta_{1t}(\theta,\alpha_t) + sq2_{1t}(\theta,\alpha_t) & \left(y_{1t}(\theta,\alpha_t)\cdot s\xi_{1t}(\theta,\alpha_t) - sk_{1t}(\theta,\alpha_t)\right) \\ x_{1t}(\theta,\alpha_t)\cdot s\eta_{1t}(\theta,\alpha_t) - sk_{1t}(\theta,\alpha_t) & -x_{1t}(\theta,\alpha_t)\cdot s\xi_{1t}(\theta,\alpha_t) + sp2_{1t}(\theta,\alpha_t) \end{bmatrix}$$

$$\mathbf{S_s}(\theta,\alpha) := \frac{1}{E\cdot I_{33}}\cdot\begin{bmatrix} -y_{1s}(\theta,\alpha)\cdot s\eta_{1s}(\theta,\alpha) + sq2_{1s}(\theta,\alpha) & \left(y_{1s}(\theta,\alpha)\cdot s\xi_{1s}(\theta,\alpha) - sk_{1s}(\theta,\alpha)\right) \\ x_{1s}(\theta,\alpha)\cdot s\eta_{1s}(\theta,\alpha) - sk_{1s}(\theta,\alpha) & -x_{1s}(\theta,\alpha)\cdot s\xi_{1s}(\theta,\alpha) + sp2_{1s}(\theta,\alpha) \end{bmatrix}$$

$$\mathbf{S_1}(\theta) := \mathbf{S_t}(\theta,\alpha_A) + \mathbf{S_s}(\theta,\psi_0+\alpha_A)$$

$$\mathbf{R}'(\theta) := \mathbf{S_1}(\theta)^{-1}\cdot\begin{pmatrix} x_V(\theta) - x_{1s}(\theta,\psi_0+\alpha_A) \\ y_V(\theta) - y_{1s}(\theta,\psi_0+\alpha_A) \end{pmatrix} \qquad \mathbf{R}'(\theta_0) = \begin{pmatrix} -1.111\times 10^{-4} \\ 7.126\times 10^{-6} \end{pmatrix} N \qquad \left|\mathbf{R}'(\theta_0)\right| = 1.113\times 10^{-4}\, N$$

$$\boldsymbol{\Delta z_t} := \mathbf{S_t}(\theta_0,\alpha_A)\cdot\mathbf{R}'(\theta_0) \qquad \boldsymbol{\Delta z_s}(\alpha) := \mathbf{S_s}(\theta_0,\alpha)\cdot\mathbf{R}'(\theta_0) + \boldsymbol{\Delta z_t}$$

$$\Delta z_{1s}(\alpha) := \boldsymbol{\Delta z_s}(\alpha)_0 + i\cdot\boldsymbol{\Delta z_s}(\alpha)_1 \qquad z_{2s}(\alpha) := z_{1s}(\theta_0,\alpha) + \Delta z_{1s}(\alpha)$$

Graphe de la déformation (2ème approximation)

Attention: Calcul long !

$$n := 20\cdot partentière(n_{sp}) + 1 \qquad i := 0..n-1 \qquad \Delta\alpha := \frac{\psi_0}{n-1} \qquad \alpha_i := i\cdot\Delta\alpha + \alpha_A$$

$$z_{2d} := \overrightarrow{z_{1t}(\theta_0,\alpha_t)} \qquad z_{2sd} := \overrightarrow{z_{2s}(\alpha)} \qquad z_{2d} := pile(z_{2d}, z_{2sd}) \qquad n_{pt} := dernier(z_{2d})$$

$$x_{2d} := Re(z_{2d}) \qquad y_{2d} := Im(z_{2d}) \qquad r_{2d} := \overrightarrow{|z_{2d}|} \qquad r_{2d_{n_{pt}}} = 0.55\, mm$$

$\beta_d := \overrightarrow{Atan(x_{2d}, y_{2d})}$ $\beta_{d_0} = 0\,deg$ $\beta_{d_{npt}} = 180\,deg$ $mod(\alpha_V(\theta_0), 2\cdot\pi) = 180\,deg$

$x_V(\theta_0) = -0.55\,mm$ $x_{2d_{npt}} - x_V(\theta_0) = 0\,mm$ $y_V(\theta_0) = 0\,mm$ $y_{2d_{npt}} - y_V(\theta_0) = 0\,mm$